

The Capture of Negative Mesotrons in Matter

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A detailed discussion of the energy loss of negative mesotrons in matter is presented. The energy range considered is from +2000 ev to the lowest quantized orbit of the mesotron. The most important mechanism for energy loss is that of electron collisions except very near the nucleus, where radiation losses are important.

The time for the over-all process is of the order of 10^{-13} sec. in condensed matter and 10^{-9} sec. in normal air. In chemical compounds the probability of capture near the various atoms is roughly proportional to their atomic numbers.

1. INTRODUCTION

RECENTLY the significance of experimental results¹ on the capture of negative mesotrons in matter has been discussed from the point of view of the information that it gives concerning the interaction between mesotrons and nucleons. The interaction of slow negative mesotrons with matter has been described as consisting of two steps:² first, one in which the mesotron is captured in the Bohr orbit with a radius of the order of 10^{-12} cm near the nucleus; second, a step in which more typically nuclear interactions play a role during which the mesotron is destroyed by its collisions with the nearby nucleons. The present paper will be primarily concerned with the detailed description of the first step.

The chief purpose of a detailed description of the capture process is to make sure that the time required for it is short compared with the natural decay time of the mesotron ($\sim 2 \times 10^{-6}$ sec.). We propose to discuss in particular how the physical and chemical state of matter influences the capture process. In this connection we shall investigate also the relative probabilities of the capture of mesotrons near various types of nuclei in case the slowing down material is not a pure element.

Throughout the greater part of the capture process the wave-length of the mesotron is short as compared to the geometric dimensions of the field in which this particle is moving. It is therefore permissible in most of our arguments to consider the motion of the mesotron as purely classical.

¹ M. Conversi, E. Pancini, and O. Piccioni, *Phys. Rev.* **71**, 209 (1947).

² E. Fermi, E. Teller, and V. Weisskopf, *Phys. Rev.* **71**, 314 (1947).

2. ENERGY LOSS OF ELECTRONS OVER 2000 EV

As long as the energy of the mesotron is more than 2000 ev, the velocity of the mesotron is greater than the velocity of the valence electrons. The slowing down of the mesotrons can then be treated according to the conventional methods applicable to fast heavy particles. In the slowing down of the mesotron the longest time is spent in the state when the mesotron moves with relativistic velocities. The consequences of the decay of the mesotron during this phase of cosmic radiation are well known and will not be discussed here. The time needed to slow a mesotron from the relativistic 10^8 volts to 2000 ev is about 10^{-9} to 10^{-10} second in condensed matter, or 1000 times as long in air. This part of the slowing process is again not our primary concern. It will be found that the time involved is considerably longer than the time needed for the later parts of the capture process. The probability of spontaneous decay during this phase, which corresponds to a range in condensed matter of a few centimeters, is only of the order of 10^{-4} . Consequently a negligible fraction of the decays observed with the ordinary experimental arrangements can be attributed to it.* Most of this time of 10^{-10} second is spent in the phases when the mesotrons still have velocities close to the light velocity. Actually the formula for energy loss per unit time is

$$-\frac{dW}{dt} = \frac{4\pi e^4 NZ}{mV} \ln\left(\frac{b_{\max}}{b_{\min}}\right). \quad (1)$$

Here W is the energy of the mesotron, V its

* Decays during this phase are completely eliminated if the observations are carried out by delayed coincidence.

velocity, m the electron mass, N is the number of atoms with atomic number Z per cubic centimeter; b_{\max} and b_{\min} are the extreme values of the collision parameters. The logarithmic factor decreases during this interval of time from a value of the order 10 to zero. The latter value is reached when the mesotron velocity becomes equal to the velocity of an electron. Thus the contributions of successive electrons vanish. Finally, the velocity of the mesotron drops below that of the valence electrons. From then on we shall discuss the process of further energy loss in detail.

3. LOSS OF ENERGY TO A DEGENERATE ELECTRON GAS

When the energy of the mesotron has dropped below 2000 eV, and its velocity is therefore less than the velocity of the valence electrons, formula (1) no longer represents a useful approximation, and the loss of energy to electrons can better be approximated in the following way.

We consider the mesotron moving inside a degenerate electron gas with a velocity V much smaller than the maximum velocity v_0 of the electrons. In this case we can estimate the energy loss as follows: in an individual collision between an electron and the mesotron, the change in speed of the electron will be of the order of magnitude V . Indeed for a head-on collision it would be $2V$. Since the electrons belong to a degenerate gas, it is clear that all the collisions for which the final velocity of the electron lies inside the occupied zone of the velocity space will be forbidden on account of the Pauli principle. Only electrons with speeds close to v_0 by amounts of the order of V will, therefore, be capable of colliding. Their number per unit volume is of the order of magnitude

$$n \approx m^3 v_0^2 V / \hbar^3. \quad (2)$$

The collision cross section σ for collisions in which the deflection is appreciable is, on the other hand, of the order of magnitude:

$$\sigma \approx (e^2 / mv_0^2)^2. \quad (3)$$

The energy transferred in collisions of this type not forbidden by the Pauli principle will always be positive and of the order of magnitude

$$W \approx mv_0 V. \quad (4)$$

From (2, 3, and 4) we can calculate the order of magnitude of the energy loss per unit time:

$$-(dW/dt) \approx W \sigma n v_0 \approx m^2 e^4 V^2 / \hbar^3 \\ \approx m^2 e^4 T / (\mu \hbar^3) \approx T / t_0, \quad (5)$$

where T is the kinetic energy of the mesotron, $t_0 = \mu \hbar^3 / m^2 e^4 = 4.84 \times 10^{-15}$ sec., and μ is the mass of the mesotron. We have set $\mu = 200m$.

The difference in the velocity dependence of the energy loss according to (1) and (5) which hold, respectively, for high and low velocities of the mesotron should be noticed. For high velocities the energy loss per unit time is inversely proportional to the velocity of the mesotron; for low velocities it is directly proportional to the square of the velocity. There is, therefore, a maximum in the energy loss which is found near the boundary of validity of the two formulae; namely, for mesotron velocities of the order of the electron velocity v_0 . One might wonder why the energy loss (5) is independent of the density of the degenerate electron gas through which the mesotron moves. Actually the collisions occur between the mesotrons and the fastest electrons, and the collision cross section decreases as $1/v_0^4$. This strong dependence on v_0 just suffices to cancel the effect of great electron density, great energy loss per collision, and great relative velocity of the colliding particles. Naturally, if (5) were taken strictly, one would obtain the absurd result that the energy loss remains unchanged even when the density of the electrons becomes extremely small.

Actually there are two reasons that limit the validity of (5) for low electron density. One is expressed by the condition

$$V \ll v_0 \quad (6)$$

that the mesotron should move slowly with respect to the electrons. The second is due to the fact that when a negative mesotron moves through an electron gas, the density of electrons near it is reduced by the electrostatic repulsion. This rarefaction of the electrons near the mesotron effectively neutralizes its charge at distances of the order:

$$(a\hbar/mv_0)^{\frac{1}{2}}, \quad (7)$$

where a is the Bohr radius

$$\hbar^2 / me^2. \quad (8)$$

The quantitative treatment of the electron-meson collision can be carried out according to the Born approximation method. This is justified if the formula

$$e^2/\hbar v_0 < 1 \quad (9)$$

holds. Now if the Born approximation is applicable, the region in which the relevant collisions take place has dimensions equal to the deBroglie wave-length of the scattered particle. In order that expression (5) be applicable we must demand that this wave-length be less than the length given in (7). It follows that

$$\hbar/mv_0 < (a\hbar/mv_0)^{\frac{1}{3}} \quad \text{or} \quad mv_0/\hbar > 1/a. \quad (10)$$

The left-hand side of the last inequality is approximately equal to the cube root of the density of the electron gas. Condition (10) means, therefore, that the density of the electron gas must be such that, on the average, more than one electron is found in a cube having a side equal to the Bohr radius. Inequalities (9) and (10) are identical in content. It is easy to show that if they are not fulfilled (5) does not apply, and the mesotron loses energy at a much slower rate than the one given by (5). Conditions (6) and (10) are independent, and the more restrictive of the two will apply. Condition (10) is usually fulfilled approximately in condensed matter of not too small density. In the case of gases, however, it will be satisfied only within the atoms, and therefore the energy loss will be confined to these regions.

Quantitatively, the energy loss of a slow mesotron in an ideal degenerate electron gas is expressed by the following integral:

$$\frac{me^4 V^2}{8\pi\hbar^3} \int_0^\pi \int_\theta^\pi \int_0^{2\pi} \frac{(\cos\theta - \cos\theta') \sin\theta d\theta \sin\theta' d\theta' d\varphi}{\sin^4(\frac{1}{2}\psi)}. \quad (11)$$

The meaning of the integration variables is the following: θ and θ' are the angles between the directions of initial and final electron velocity and the direction of the mesotron velocity; ψ is the angle of deflection for the electron; and φ is given by

$$\cos\psi = \cos\theta \cos\theta' + \sin\theta \sin\theta' \cos\varphi. \quad (12)$$

The integral diverges logarithmically near $\psi=0$. The divergence can be removed by taking into account the fact that collisions involving a small value of ψ occur at large distances where the mesotron charge is screened.

Collisions at a distance greater than (7) will not contribute, and one needs to consider only momentum changes greater than

$$(\hbar mv_0/a)^{\frac{1}{3}}, \quad (13)$$

which is \hbar divided by the length (7). From this one finds that the integration need be carried down only to a value

$$\psi_{\min} = (\hbar mv_0/a)^{\frac{1}{3}} (mv_0)^{-1} = (e^2/\hbar v_0)^{\frac{1}{3}} = (c/137v_0)^{\frac{1}{3}}. \quad (14)$$

One should notice that if (10) is fulfilled, ψ_{\min} is small compared with unity, as it should be. Eliminating the divergence with this prescription, one can evaluate (11) and find finally the following estimate for the rate of energy loss:

$$\begin{aligned} -\frac{dW}{dt} &= \frac{4}{3\pi} \frac{m^2 e^4 V^2}{\hbar^3} \ln \frac{1}{\psi_{\min}} \\ &= \frac{2}{3\pi} \frac{m^2 e^4 V^2}{\hbar^3} \ln \frac{137v_0}{c}, \end{aligned} \quad (15)$$

or finally,

$$-\frac{dW}{dt} = \frac{4}{3\pi} \frac{m^2 e^4 T}{\mu \hbar^3} \ln \frac{137v_0}{c}. \quad (16)$$

The logarithm will be of a small numerical order of magnitude and will have somewhat larger values in the deep portions of the atom. For estimates of the order of magnitude it will be permissible to use formula (5).

The previous theory will be applicable to those cases in which the electrons in the vicinity of the mesotron can properly be described as a degenerate gas. In particular, the theory will break down within the K shells of the atoms, since the electron density there is less than the value corresponding to the degenerate gas. We shall return to this question in the next section. We shall have to give special consideration to the case of insulators and gases where electrons may be excited only by discontinuous amounts. The case that most nearly approaches the ideal conditions is that of the metals, which we shall now discuss.

4. METALS

In order to calculate the energy loss as a function of the mesotron energy, we have to calculate the average value of the kinetic energy \bar{T} which is to be substituted in formula (5). In calculating average values of T , one makes use of the fact that the probability of finding the mesotron in a given volume element is weighted by the square root of the kinetic energy that the mesotron has at this position. The reason for this weighting is that the volume in the momentum space available to the mesotron is proportional to

$$T^{\frac{1}{2}}dT = T^{\frac{1}{2}}dW. \quad (17)$$

The average kinetic energy is therefore:

$$\bar{T} = \frac{\int (W-U)^{\frac{1}{2}}d\tau}{\int (W-U)^{\frac{1}{2}}d\tau}, \quad (18)$$

where U is the potential energy and $d\tau$ is the volume element.

For high mesotron energy U will be negligible compared with W , and \bar{T} will be equal to W . As the energy approaches zero, namely, that value for which the mesotron can no longer freely pass from one atom to another, the average kinetic energy becomes appreciably larger than W because U is negative. For W negative, the mesotron is bound to a definite atom. For negative values of energy the kinetic energy is of the order of magnitude of the absolute value of W , as will be discussed later.

For positive W values one obtains a low limit for \bar{T} and the energy loss replacing $(W-U)^{\frac{1}{2}}$ by the smaller expression $W^{\frac{1}{2}}+(-U)^{\frac{1}{2}}$ in the numerator of the integral, and replacing $(W-U)^{\frac{1}{2}}$ by the larger expression $W^{\frac{1}{2}}+(-U)^{\frac{1}{2}}$ in the denominator. The error caused by these substitutions is a maximum when W is equal to the absolute value of U , and is then a factor 2. We write thus

$$\bar{T} \approx \frac{\int W^{\frac{1}{2}}d\tau + \int (-U)^{\frac{1}{2}}d\tau}{\int W^{\frac{1}{2}}d\tau + \int (-U)^{\frac{1}{2}}d\tau}. \quad (18a)$$

We shall use the potential obtained from the statistical model

$$U = -\frac{Z^{4/3}e^2}{b} \frac{\varphi(x)}{x}, \quad (19)$$

where we have set for the distance from the nucleus $r = xbZ^{-\frac{1}{3}}$ and the length b is

$$b = (9\pi^2/128)^{\frac{1}{3}} \times \text{Bohr radius} \\ = 0.47 \times 10^{-8} \text{ cm.} \quad (20)$$

The function φ has been tabulated.³ In some of the following calculations we use the crude approximation**

$$\varphi = 0.4/x. \quad (21)$$

Approximating a lattice cell by a sphere one obtains \bar{T} by integrating (18a) over the cell. The second integral in the numerator can be performed by partial integration using the differential equation³ for φ . One finds

$$\bar{T} = \frac{N^{-1}W^{\frac{1}{2}} + 4\pi e^3 b^{\frac{1}{2}} Z [1 - \varphi(x_0) + x_0 \varphi'(x_0)]}{N^{-1}W^{\frac{1}{2}} + 4\pi e b^{\frac{5}{2}} Z^{-\frac{1}{3}} \int_0^{x_0} \varphi^{\frac{1}{2}} x^{\frac{1}{2}} dx}. \quad (22)$$

Here N is the number of atoms per cubic centimeter, and x_0 is the value of x at the edge of the cell. Using (21) we get

$$\bar{T} = \frac{N^{-1}W^{\frac{1}{2}} + 4\pi Z e^3 b^{\frac{1}{2}} (1 - 0.8/x_0)}{N^{-1}W^{\frac{1}{2}} + 3.96 e b^{\frac{5}{2}} Z^{-\frac{1}{3}} x_0^2}, \quad (23)$$

where x_0 is given by the relation:

$$1/N = 4\pi b^3 x_0^3 / 3Z. \quad (24)$$

Values of \bar{T} as obtained from (23) are given in Table I for graphite and iron. One finds that \bar{T} has a flat minimum at 7 ev and 20 ev, respectively. This will be, therefore, the value of the energy at which energy is lost at the slowest rate. At higher energies Table I shows that \bar{T} becomes less than W . This is because of the approximation which we made in substituting (18) by (18a). Actually $\bar{T} > W$ always holds, and we underestimate \bar{T} if we use $\bar{T} = W$ for high values of W . From Table I and formula (5), one can calculate, for the two cases in question,

³ E. Fermi, Zeits. f. Physik 48, 73 (1928).

** From $x=0.5$ up to almost $x=8$ the quantity $x\varphi$ remains between the limits 0.3 and 0.5.

the time needed for the mesotron to lose energy from 2000 ev to zero. One finds 2.6×10^{-14} sec. in graphite and 2.2×10^{-14} sec. in iron. Somewhat longer times would be found in condensed matter of lower density. As a practical average time for crossing the interval from 2000 ev to zero we take for all types of condensed matter about 3×10^{-14} sec.

We proceed now to the question of energy loss when W is negative, when the mesotron can be considered bound to a special atom. If the energy is negative and its absolute value is sufficiently large, it is a sufficient approximation to set the kinetic energy

$$\bar{T} = \alpha |W|, \quad (25)$$

where α is a number of the order of unity. (Actually $\alpha=1$ for a Coulomb field and $\alpha>1$ for the statistical potential.) Expression (25) will lead to a very small loss of energy for small absolute values of W . Actually at $W=0$ the kinetic energy does not vanish, and may be obtained from (23):

$$\bar{T}(0) = \frac{3.2e^2Z^{4/3}}{bx_0^2}(1 - 0.8/x_0). \quad (26)$$

Since at $W=0$, \bar{T} increases with decreasing W we may use (26) as a lower limit of \bar{T} at negative energies. We shall use for the energy-loss expression (5), and substitute for \bar{T} expression (25) or (26), whichever is the greater. Expression (26) will be relevant from $W=0$ to $-W \approx 50$ ev. The time required to cross this energy region is of the order of $t_0 = 4.84 \times 10^{-15}$ sec. In the range where \bar{T} is estimated by (25), W as a function of time is given by

$$-W \approx \bar{T}(0)e^{\alpha t/t_0}. \quad (27)$$

This formula is valid as long as the statistical model is permissible, i.e., as long as the mesotron moves outside of the radius of the K -shell. At distances smaller than this radius the actual electron density is less than the density obtained from the statistical model. Nevertheless the energy loss of the mesotron continues to proceed according to the formulas (16), (5), and (27) even when the mesotron is somewhat closer to the nucleus than the radius of the K -shell. The reason for this is that the energy loss of the mesotron does not depend on the electron den-

TABLE I. Average kinetic energy as a function of the total energy.

$W(\text{ev})$	$\bar{T}(\text{ev})$	
	Graphite	Iron
0	36	86
5	23	62
10	24	56
20	27	55
30	32	57
50	45	65
100	83	95

sity, and thus the failure of the statistical model to predict the correct density within the K -shell has no direct effect on the behavior of the mesotron.

When the mesotron moves inside the K -shell it is not permissible, however, to apply the method of deriving formulas (16) and (5) which we have given above. In a collision the energy mv_0V is exchanged, and a time not shorter than \hbar/mv_0V is needed to describe such a collision. During this time the mesotron moves through a distance \hbar/mv_0 . Our discussion so far has assumed that during a collision the mesotron moves in a straight line. When the mesotron moves along the K -orbit of the atom its path has a radius of curvature equal to \hbar/mv_0 . For orbits on or within the K -shell it will be a better approximation to consider the time-dependent dipole consisting of the mesotron and a unit positive charge located on the nucleus. One may then calculate, by applying perturbation theory to the statistical model, the energy exchange between the mesotron and the electrons.

If one assumes that the mesotron moves on a circular orbit one may evaluate the rate of energy loss dW/dt . The result of the calculation is identical with (16) provided that the minimum momentum change of the electrons $(\hbar mv_0/a)^{\frac{1}{2}}$ obtained from (7) is greater than the minimum momentum change $\hbar\omega/v_0$ compatible with the requirement that a mesotron with angular velocity ω exchanges energy with the electrons in quanta $\hbar\omega$. At radii somewhat smaller than the radius of the K -shell the inequality $\hbar\omega/v_0 > (\hbar mv_0/a)^{\frac{1}{2}}$ holds for not too light atoms. Then (16) must be replaced by

$$-\frac{dW}{dt} = \frac{8e^4m^2T}{3\pi\mu\hbar^3} \ln\left(\frac{mv_0^2}{\hbar\omega}\right). \quad (28)$$

Expression (5) remains a satisfactory approximation as long as the logarithmic factor does not become small compared to unity, i.e., as long as the energy $\hbar\omega$ transferred to the electrons is smaller than the maximum kinetic energy $mv_0^2/2$ of the electrons. For $mv_0^2 \ll \hbar\omega$ the perturbation calculation gives

$$-\frac{dW}{dt} = \frac{2^{5/2}\pi m^3 n_0 e^4 r^2}{3\hbar^3} \left(\frac{Ze^2}{\mu r^3}\right)^4, \quad (29)$$

where n_0 is the electron density near the nucleus and r is the radius of the circular orbit of the mesotron. Expression (29) agrees with the results of calculations on internal conversion, for the non-relativistic case, provided that the energy of the ejected electron is great compared with the energy of the K -electrons.⁴ It is of interest to note that (28) holds if the radius r of the mesotron orbit is greater than

$$r_1 = (m/\mu)^{1/3} r_K = 0.171 r_K. \quad (30)$$

Here r_K is the radius of the K -shell. For $r < r_1$ expression (29) is valid.

For very small values of r , energy loss by radiation is faster than energy loss due to interaction with electrons. For circular mesotron orbits one finds by comparing (29) with the radiated energy that radiation becomes predominant for $r < r_2$, where

$$r_2 = 4^{-1/7} (m/\mu)^{1/3} (\hbar c/e^2)^{3/7} Z^{-5/21} r_c \\ = (4.5/Z^{5/21}) \times 10^{-11} \text{ cm}, \quad (31)$$

where $r_c = \hbar/mc$ is the Compton wave-length divided by 2π .

From (29) one can calculate the time needed to

TABLE II. Summary of slowing-down times in graphite and iron.

Energy range (ev)	Time	
	Graphite	Iron
2000 → 0	5.4 t_0	4.5 t_0
0 → $-Ze^2/r_1$	5.3 t_0	7.4 t_0
$-Ze^2/r_1$ → $-Ze^2/r_2$	4.6 t_0	0.3 t_0
$-Ze^2/r_2$ → $-Ze^2/r_0$	3.6 t_0	0.3 t_0
2000 → $-Ze^2/r_0$	18.9 t_0	12.5 t_0
(total "slow" range)	9.2×10^{-14} sec.	6.1×10^{-14} sec.

⁴ See, e.g., Rasetti, *Elements of Nuclear Physics* (Prentice-Hall, New York, 1936), pp. 134-139.

drop from r_1 to r_2

$$t_{r_1 \rightarrow r_2} = \frac{1}{6\sqrt{2}} \left(\frac{m}{\mu}\right)^{3/4} \left[\left(\frac{r_K}{r_2}\right)^{9/4} - \left(\frac{r_K}{r_1}\right)^{9/4} \right] t_0 \\ = \left[\frac{103}{Z^{12/7}} - 0.117 \right] t_0. \quad (32)$$

While the classical expression for the radiation by an accelerated charge gives the time spent between r_2 and the lowest quantum orbit $r_0 = \hbar^2/Z\mu e^2$,

$$t_{r_2 \rightarrow r_0} = \frac{\mu^2 c^3}{4e^4 Z} (r_2^3 - r_0^3) \\ = \frac{\mu}{4mZ} \left[\left(\frac{r_2}{r_c}\right)^3 - \left(\frac{r_0}{r_c}\right)^3 \right] t_0 \\ = \left[\frac{78}{Z^{12/7}} - \frac{16}{Z^4} \right] t_0. \quad (33)$$

For heavy nuclei r_1 becomes smaller than r_2 , and there is no region in which (29) determines the energy loss. Further consequences of high Z -values are that r_0 becomes less than the nuclear radius and that pair production begins to play a role in the last steps of the slowing-down process. All these effects shorten the time the mesotron needs to get close to the nucleus.

For carbon and iron, Table II summarizes the times (measured in units $t_0 = 4.84 \times 10^{-15}$ sec.) the mesotron needs to cross the energy regions indicated in the first column of the table.

One sees that the slowing down time is less than 10^{-13} sec. This is very short compared with the lifetime of the mesotron, 2×10^{-6} sec.

5. INSULATORS

The case of insulators differs from that of metals because the amount of energy that may be delivered to electrons in a metal can be arbitrarily small, whereas in an insulator it must be at least as large as the gap between two Brillouin zones. This usually amounts to several volts. The loss of energy to electrons will be thereby reduced in those cases in which energy is transferred in small individual amounts.

In a collision the amount of energy transferred to the electron is of the order of magnitude $mv_0 V$,

and we can expect that Eq. (5) will hold only if

$$mv_0V > G, \quad (34)$$

where G is the "Brillouin gap," i.e., the minimum energy that the electrons can accept. In case (34) is not fulfilled, the rate of energy loss will be smaller. This limitation will necessitate a change in formula (18) where the integral in the numerator will no longer be extended to all portions of space accessible to the mesotron but only to those for which in addition (34) is fulfilled. For positive W , the effect on dW/dt is most important at and near $W=0$. For $Z \geq 6$ actual calculation based on the statistical atom model shows that the rate of energy loss is not changed in this critical region by more than a factor $\frac{1}{2}$. The corresponding increase in the slowing-down time is less than 10^{-14} sec.

For positive energies, regions close to nuclei where (34) is fulfilled will occasionally be visited by the mesotron. For negative energies regions close to the nucleus may be avoided if the mesotron is captured into an almost circular orbit. Therefore the possibility arises of the mesotron spending a long time in such a circular orbit. We shall show that actually the mesotron will spend in a circular orbit a time which is not long compared to the other times we have obtained in the slowing-down process.

Circular orbits of negative energy exist only at a considerable depth within the atom. This can be proved using the statistical potential (19). Circular orbits of negative energy exist only where $x\varphi(x)$ is an increasing function of x .*** The value of $x\varphi(x)$ is zero at $x=0$, increases to a maximum reached at $x=2.25$, and decreases beyond this point. Consequently, circular orbits of negative energy are to be found only within a distance from the nucleus corresponding to $x=2.25$; namely,

$$r = 2.25b/Z^{\frac{1}{3}}. \quad (35)$$

For a circular orbit at exactly this radius the energy is equal to zero, and the kinetic energy of the mesotron is equal to

$$0.09e^2Z^{4/3}/b = 2.7Z^{4/3} \text{ ev}. \quad (36)$$

*** This statement does not refer to circular orbits outside the core of an atom. In condensed matter there is usually no room available for orbits of such very large radius.

If a mesotron is captured into an orbit which is not circular, its closest approach to the nucleus will be even smaller than the distance given by formula (35), and the maximum kinetic energy will be larger than that given by (36). Applying condition (34), we find that energy loss to the electrons will be possible from at least one part of the orbit if the condition

$$0.09e^2Z^{4/3}/b \text{ ev} > \frac{1}{2}(\mu/m)^{\frac{1}{2}}G \quad (37)$$

is fulfilled. If we set for G the fairly large value of 7 volts, we see that relation (37) is fulfilled if Z is 9 or greater. Actually relations (34) and (37) are not to be taken in a quite strict sense because head-on collisions between electrons and mesotrons will give an energy exchange twice as large as assumed in (34). If this is taken into account an additional factor of $\frac{1}{2}$ appears in the right-hand side of (34). If we again assume 7 volts for G the limiting value of Z drops from 9 to 6.

Stable circular orbits of positive energy exist for x values greater than 2.25. The condition of stability for a circular orbit is:

$$\varphi - x\varphi' - x^2\varphi'' > 0. \quad (38)$$

This condition is fulfilled up to $x=3.3$, or

$$r = 3.3bZ^{-\frac{1}{3}}. \quad (39)$$

A mesotron moving on a circular orbit of this radius has a smaller velocity than one moving on the radius given by (35), and one may expect increased difficulties in the energy exchange between mesotron and electron. Actually, in light atoms the greatest stable circular radius (39) differs from (35) by less than the uncertainty due to the spread of the wave function of the mesotron. At the same time the difference in angular momenta between the greatest circular orbit and (35) is small. For $Z=6$ this difference is less than \hbar . Thus a mesotron moving in a stable orbit of positive energy will not lose energy at a much smaller rate than will a mesotron whose wave function has its maximum at the radius given by (35).

We can conclude that the circular orbits will hardly ever be too stable. Even in case they should be formed around an element like lithium or beryllium the total time of energy loss will probably be less than the lifetime of a mesotron.

Furthermore, the actual size of the Brillouin gap is affected by the localization of the mesotron

on one lattice atom. Since, at least in the critical cases, the mesotron is captured fairly far inside the atom, the atom is effectively turned into an element with atomic number $(Z-1)$. If we were dealing with an isolated atom, this would lower the ionization energy of the atom and turn it into the much smaller value which usually is called the electron affinity of the atom of charge $(Z-1)$. Actually this electron affinity may even be zero. In the special case of mesotron capture by the hydrogen atom, it is found that when the mesotron approaches the nucleus to a distance of 0.639 Bohr radii, the binding energy of the electron becomes zero. In the closed shell structures usually found in insulators, an electron affinity of two or three volts is likely to remain. As a consequence of this the mesotron, after it is captured, may lose as a first step an energy smaller than the Brillouin gap. After this loss the atom in which the mesotron is now localized does not have a closed shell, and further excitation may still require less energy than the width of the Brillouin gap. Of course, further ionizations would tend to raise the ionization potential, but the local electron deficiency will be promptly filled by capturing electrons from neighbors.

A special situation arises when the mesotron is captured on a hydrogen atom, as may happen for instance in paraffin. In that case the hydrogen and the mesotron circulating around it form a small neutral system which will move along and will readily permeate to any part of the lattice. As a result one will expect that the mesotron will eventually be caught in the field of a more highly charged nucleus.

After the mesotron has attained negative energies of an absolute value greater than 100 ev, the further energy loss proceeds in insulators as it does in metals.

In conclusion we see that the total time needed for energy loss in insulators is apt to be a little longer than in metals, because of the difficulty in bridging the Brillouin gap. There will be, however, no change in the order of magnitude of the total time which the mesotron needs to reach its lowest orbit.

6. GASES

In gases, as in insulators, electrons cannot accept from the mesotron arbitrarily small

amounts of energy. The lowest electronic excitation energy, I , of a gas molecule plays the same role as the Brillouin gap does in insulators.

For positive values of W the energy loss proceeds according to (5) and (18). The upper limit of the integrand in the numerator is determined by the condition

$$mv_0V = I. \quad (40)$$

The integral in the denominator is extended over the whole space. As a result the average value of the energy loss $-dW/dt$ is reduced by the ratio of gas density to insulator density. Under conditions of normal temperature and density the energy loss in a gas is about a thousand times smaller than in a solid insulator. The time needed to slow down the mesotrons from $W=2000$ ev to $W=0$ is approximately 3×10^{-11} seconds.

For negative values of W the mesotron is localized on a specific molecule. As the energy transfer to electrons proceeds, it is likely that enough energy will be given to nuclear motion to cause dissociation of the molecule on which the mesotron is found. Thus we may confine our attention to the atom which carries the mesotron along. The energy loss of the mesotron causes progressive ionization of this atom, and, as a result, the minimum excitation energy I is increased. This may lead to an effective stoppage of the energy loss if the mesotron happens to be moving on a nearly circular orbit. The energy loss may stop when the relation

$$\hbar\omega \approx I \quad (41)$$

is fulfilled. Here ω is the frequency of the mesotron in a circular orbit.

Further energy loss of the mesotron may thus be delayed until the ion carrying the mesotron makes a collision with another molecule. Then a substantial part of the ionic charge will be neutralized, and the energy transfer from the mesotron to the electron cloud will be resumed. We assume that the excitation energy I of the atom which carries the mesotron may be written in the form

$$I = Ki^2, \quad (42)$$

where i is the degree of ionization and K is a constant. The total energy E needed to raise the

ionization from zero to i is

$$E = \frac{1}{3}K\dot{i}^3. \quad (43)$$

We assume that between two collisions of the mesotron-carrying atom the energy of the mesotron W will change by $E/2$. We have then

$$-\frac{dW}{dt} = \frac{1}{2}NE = \frac{1}{6}NK\dot{i}^3, \quad (44)$$

where N is the number of collisions per unit time $\sim 10^{10} \text{ sec}^{-1}$.

The energy loss of the mesotron will be smallest if the mesotron continues to move on circular orbits. For the radius r of this orbit we write as previously

$$r = xbZ^{-\frac{1}{2}}. \quad (45)$$

Using (19), (21), and (41) we get

$$I = \frac{heZ}{\mu^{\frac{1}{2}}b^{\frac{3}{2}}x^{\frac{3}{2}}} \left[-\frac{d}{dx} \left(\frac{\varphi}{x} \right) \right]^{\frac{1}{2}} = 2.06Zx^{-2} \text{ ev}. \quad (46)$$

From (42), (43), and (44) we now obtain

$$-\frac{dW}{dt} = 0.493NK^{-\frac{1}{2}}(eV)^{\frac{3}{2}}Z^{\frac{3}{2}}x^{-3}. \quad (47)$$

If W is calculated for a circular orbit from the statistical model one obtains

$$W = -\frac{Z^{4/3}e^2}{2bx} d(x\varphi)/dx. \quad (48)$$

Here the approximation (21) is not sufficient. In the region where circular orbits of negative energy are possible we may use for the expression $x\varphi$

$$x\varphi = 0.489 - 0.025(x - 2.25)^2 + 0.015(x - 2.25)^3. \quad (49)$$

For $x < 0.5$, expression (49) is not a good approximation. This region, however, contributes little to the slowing-down time and (49) suffices, therefore, for our purpose.

We calculate the total time by integrating

$$\begin{aligned} t &= \int_{2.25}^0 \frac{dt}{dx} dx = \int_0^{2.25} \left[\frac{dW}{dx} / \left(-\frac{dW}{dt} \right) \right] dx \\ &= \frac{1}{N} \int_0^{2.25} \frac{Z^{4/3}e^2(2b)^{-1}[(0.34/x^2) - 0.045]}{0.493Z^{\frac{3}{2}}(eV)^{\frac{3}{2}}K^{-\frac{1}{2}}x^{-3}} dx \\ &= 15.5 \left(\frac{K}{eV} \right)^{\frac{1}{2}} \frac{Z^{-1/6}}{N}. \end{aligned} \quad (50)$$

A reasonable value of K is 5 ev. This gives

$$t = 35 \frac{Z^{-1/6}}{N}. \quad (51)$$

The total time needed for the energy-loss process at negative energies will therefore be of the order of 10^{-9} second even if the mesotron continues to move on circular orbits.

7. CHEMICAL COMPOUNDS

It is of interest in experiments on the disappearance of negative mesotrons in chemical compounds to find out with what relative probability the mesotron is captured by the different kinds of atoms. We are led by crude estimates to the conclusion that the capture probability is proportional to the nuclear charge Z . This may be seen as follows.

It is simplest to set the capture probability proportional to the energy loss of the mesotrons near the various atomic species. At low positive mesotron energy, which is the relevant region for our argument, the energy loss is given by an expression whose numerator contains the numerator of (23). For $W=0$ this is proportional to Z . The denominator is a constant integral for all atomic species and does not enter in the evaluation of the ratio of energy losses.

The capture probability will actually be proportional to the rate of energy loss only if the ratio of these rates does not change too rapidly near $W=0$. In particular it is necessary to demand that this ratio should not be strongly altered by the energy change due to a single passage of the mesotron through an atom. For this energy loss δW we may write

$$\delta W = \int [\frac{1}{2}\mu(W-U)]^{\frac{1}{2}} ds/t_0. \quad (52)$$

Near $W=0$ we get

$$\delta W = 2.3eVZ^{\frac{1}{2}} \ln(x_0/x_{\min}) \quad (53)$$

where x_{\min} corresponds to the distance of minimum approach. The energy given in (53) is small enough so that it does not yet affect significantly the numerator in (23). Thus the ratios of energy losses are hardly affected by δW , and we may

conclude that the ratios of capture probabilities are proportional to Z .

8. CONCLUSION

The over-all conclusions can be summarized as follows. In condensed substances, both conductors and insulators, a negative mesotron is captured in its orbit nearest to the nucleus in about 10^{-13} second. In a gas the corresponding

time is a little longer than is indicated by the ratio of densities. In particular, in normal air it is of the order of 10^{-9} sec. In both cases this time is very short compared with the mesotron natural lifetime of 2×10^{-6} sec. so that the mode of ultimate disappearance of the negative mesotron is governed by the balance between natural decay and typically nuclear phenomena leading to mesotron disappearance.

Spin Dependence of Scattering of Slow Neutrons by Be, Al, and Bi

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Some information has been obtained on the spin dependence of scattering of slow neutrons by Be, Al, and Bi by measuring the scattering cross section for filtered neutrons. The result is that in none of these three cases does the sign of the scattering length change when the spin orientation is changed. But in the case of Be and Bi the magnitude of the scattering length for one spin orientation may be up to twice as great as that for the other spin orientation, and in the case of Al the variation may be by a factor of three.

SOME information on the spin dependence of the scattering of slow neutrons can be obtained by measuring the cross sections of some microcrystalline substances for filtered neutrons.^{1,2}

When a slow neutron is scattered by an atom having nuclear spin I , two values for the scattering length² can be expected according to whether the spin of the neutron is parallel or antiparallel to I ; these will be indicated by a_+ and a_- . If these two values are equal, there is no spin dependence of the scattering. In this case interference phenomena are not influenced by the spin, and the neutron waves scattered by the atoms behave as fully coherent. When a_+ and a_- are different, the coherent scattering of the atom is determined by an average scattering length: (see reference 2, formula (6))

$$a = \frac{I}{2I+1} a_- + \frac{I+1}{2I+1} a_+$$

The remaining scattering behaves as incoherent for interference phenomena.

In order to discuss the significance of coherent and incoherent scattering it is necessary to distinguish between collisions in which the spin orientation is not changed and those in which it changes. The first type of collision is responsible for coherent scattering, the second for incoherent. The reason is that interference takes place only when the scattering is due to the cooperative action of all atoms. This is the case when there is no spin change to indicate which atom has been responsible for the scattering. If there is a spin change, however, the scattering is attributed to the individual action of that atom whose spin has changed.

One can prove by elementary quantum mechanics that the coherent scattering cross section is

$$\sigma_{\text{coherent}} = 4\pi \left\{ \frac{I}{2I+1} a_- + \frac{I+1}{2I+1} a_+ \right\}^2, \quad (1)$$

¹ H. L. Anderson, E. Fermi, and L. Marshall, Phys. Rev. **70**, 815 (1946).

² E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947).